

1. An array is passed by pointer. Time =  $\Theta(1)$ .
  2. An array is passed by copying. Time =  $\Theta(N)$ , where  $N$  is the size of the array.
  3. An array is passed by copying only the subrange that might be accessed by the called procedure. Time =  $\Theta(q - p + 1)$  if the subarray  $A[p..q]$  is passed.
- a. Consider the recursive binary search algorithm for finding a number in a sorted array (see Exercise 2.3-5). Give recurrences for the worst-case running times of binary search when arrays are passed using each of the three methods above, and give good upper bounds on the solutions of the recurrences. Let  $N$  be the size of the original problem and  $n$  be the size of a subproblem.
- b. Redo part (a) for the MERGE-SORT algorithm from Section 2.3.1.

#### 4-4 More recurrence examples

Give asymptotic upper and lower bounds for  $T(n)$  in each of the following recurrences. Assume that  $T(n)$  is constant for sufficiently small  $n$ . Make your bounds as tight as possible, and justify your answers.

- a.  $T(n) = 3T(n/2) + n \lg n$ .
- b.  $T(n) = 5T(n/5) + n \lg n$ .
- c.  $T(n) = 4T(n/2) + n^2 \sqrt{n}$ .
- d.  $T(n) = 3T(n/3 + 5) + n/2$ .
- e.  $T(n) = 2T(n/2) + n \lg n$ .
- f.  $T(n) = T(n/2) + T(n/4) + T(n/8) + n$ .
- g.  $T(n) = T(n - 1) + 1/n$ .
- h.  $T(n) = T(n - 1) + \lg n$ .
- i.  $T(n) = T(n - 2) + 2 \lg n$ .
- j.  $T(n) = \sqrt{n}T(\sqrt{n}) + n$ .

#### 4-5 Fibonacci numbers

This problem develops properties of the Fibonacci numbers, which are defined by recurrence (3.21). We shall use the technique of generating functions to solve